

Supersymmetric CP^N Sigma Model on Noncommutative Superspace

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Abstract

We construct a closed form of the action of the supersymmetric CP^N sigma model on noncommutative superspace in four dimensions. We show that this model has $\mathcal{N} = \frac{1}{2}$ supersymmetry and that the transformation law is not modified. The supersymmetric CP^N sigma model on noncommutative superspace in two dimensions is obtained by dimensionally reducing the model in four dimensions.

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1 Introduction

Noncommutative geometry [1] appears in M-theory, string theory and condensed matter physics. Noncommutative field theories are known to describe the effective theory of string in a constant NS-NS B field [2]. (2+1)-dimensional noncommutative field theories have been applied to the quantum Hall effect.

In supersymmetric field theories, there are a few alternatives in introducing non(anti)commutativity of the supercoordinates $(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ ¹. In particular, supersymmetric Yang-Mills theory on *noncommutative superspace* [4, 5, 6] describes the effective field theory of string in a constant selfdual graviphoton background [6, 7, 8]. In the field theoretical view point, these theories keep $\mathcal{N} = \frac{1}{2}$ supersymmetry and have some interesting properties.

In this letter, we construct the supersymmetric nonlinear sigma model whose target space is CP^N (CP^N SNLSM) on noncommutative superspace in four and two dimensions. Low-dimensional SNLSMs on ordinary superspace have interesting properties. In two dimensions, the CP^N SNLSM is integrable, *i.e.*, it has infinitely many conservation laws. It shares important properties with four-dimensional supersymmetric gauge theories, such as asymptotic freedom and dynamical mass gap. In three dimensions, the CP^N SNLSM has been investigated using the large- N expansion [9]. As we will see below, since the Kähler potential of SNLSM is generally non-polynomial, the action of SNLSM on noncommutative superspace has infinitely many terms [10]. It is difficult to study the properties of this model either perturbatively or non-perturbatively. We introduce an auxiliary vector superfield to linearize the CP^N SNLSM, mimicking the commutative case [11]. Once introducing the vector superfield, we can eliminate all auxiliary fields and obtain a closed form of the action.

¹ We follow the notation of [3].

2 Noncommutative Superspace

2.1 Noncommutative Superspace

We recapitulate noncommutative superspace, closely following Seiberg [6]. We consider four-dimensional $\mathcal{N} = 1$ supersymmetric field theories on the noncommutative superspace. The non(anti)commutativity is introduced by

$$\begin{aligned}\{\theta^\alpha, \theta^\beta\} &= C^{\alpha\beta}, \quad \{\theta^\alpha, \bar{\theta}^{\dot{\alpha}}\} = \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0, \\ [y^\mu, y^\nu] &= [y^\mu, \theta^\alpha] = [y^\mu, \bar{\theta}^{\dot{\alpha}}] = 0,\end{aligned}\tag{1}$$

where y^μ is the chiral coordinate

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}.\tag{2}$$

The product of functions of θ is Weyl ordered by using the Moyal product, which is defined by

$$\begin{aligned}f(\theta) * g(\theta) &= f(\theta) \exp\left(-\frac{1}{2}C^{\alpha\beta}\overleftarrow{\frac{\partial}{\partial\theta^\alpha}}\overrightarrow{\frac{\partial}{\partial\theta^\beta}}\right)g(\theta) \\ &= f(\theta) \left[1 - \frac{1}{2}C^{\alpha\beta}\overleftarrow{\frac{\partial}{\partial\theta^\alpha}}\overrightarrow{\frac{\partial}{\partial\theta^\beta}} - \det C \overleftarrow{\frac{\partial}{\partial(\theta\theta)}}\overrightarrow{\frac{\partial}{\partial(\theta\theta)}}\right]g(\theta).\end{aligned}\tag{3}$$

The supercovariant derivatives are defined by

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + 2i\sigma_{\alpha\dot{\alpha}}^\mu\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial y^\mu}, \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}.\tag{4}$$

Since D_α and $\bar{D}_{\dot{\alpha}}$ do not contain θ , their anticommutation relations are same as those on the commutative superspace.

$$\{D_\alpha, D_\beta\} = 0, \quad \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0, \quad \{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma_{\alpha\dot{\alpha}}^\mu\frac{\partial}{\partial y^\mu}.\tag{5}$$

The supercharges are defined by

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha}, \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + 2i\theta^\alpha\sigma_{\alpha\dot{\alpha}}^\mu\frac{\partial}{\partial y^\mu}.\tag{6}$$

Since $\bar{Q}_{\dot{\alpha}}$ contains θ , the anticommutation relations are modified as follows.

$$\{Q_{\alpha}, Q_{\beta}\} = 0, \quad \{Q_{\alpha}, \bar{Q}_{\dot{\alpha}}\} = +2i\sigma_{\alpha\dot{\alpha}}^{\mu} \frac{\partial}{\partial y^{\mu}}, \quad (7)$$

$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = -4C^{\alpha\beta}\sigma_{\alpha\dot{\alpha}}^{\mu}\sigma_{\beta\dot{\beta}}^{\nu} \frac{\partial^2}{\partial y^{\mu}\partial y^{\nu}}. \quad (8)$$

Furthermore, $\bar{Q}_{\dot{\alpha}}$ does not act as derivations on the Moyal product of fields

$$\bar{Q}_{\dot{\alpha}}(f * g) \neq (\bar{Q}_{\dot{\alpha}}f) * g + f * (\bar{Q}_{\dot{\alpha}}g). \quad (9)$$

Then $\bar{Q}_{\dot{\alpha}}$ is not a symmetry of the theory in general, hence we have $\mathcal{N} = \frac{1}{2}$ supersymmetry.

2.2 Superfields

The chiral superfield is defined by $\bar{D}_{\dot{\alpha}}\Phi = 0$, and hence, $\Phi = \Phi(y, \theta)$. In terms of the component fields, it is given by

$$\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \quad (10)$$

where $\theta\theta = -\theta^1\theta^2 + \theta^2\theta^1$ and is Weyl ordered.

The antichiral superfield is defined by $D_{\alpha}\bar{\Phi} = 0$, and hence, $\bar{\Phi} = \bar{\Phi}(\bar{y}, \bar{\theta})$, where \bar{y}^{μ} is given by

$$\bar{y}^{\mu} = y^{\mu} - 2i\theta\sigma^{\mu}\bar{\theta}, \quad (11)$$

$$[\bar{y}^{\mu}, \bar{y}^{\nu}] = 4\bar{\theta}\bar{\theta}C^{\mu\nu}, \quad C^{\mu\nu} = C^{\alpha\beta}\epsilon_{\beta\gamma}(\sigma^{\mu\nu})_{\alpha}{}^{\gamma}. \quad (12)$$

In the component fields, it is convenient to express the antichiral superfield in terms of y and θ and to Weyl order the θ s

$$\begin{aligned} \bar{\Phi}(y - 2i\theta\sigma\bar{\theta}, \bar{\theta}) &= \bar{\phi}(y) + \sqrt{2}\bar{\theta}\bar{\psi}(y) - 2i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\bar{\phi}(y) \\ &\quad + \bar{\theta}\bar{\theta}\left[\bar{F}(y) + \sqrt{2}i\theta\sigma^{\mu}\partial_{\mu}\bar{\psi}(y) + \theta\theta\partial^2\bar{\phi}(y)\right]. \end{aligned} \quad (13)$$

We also need the $U(1)$ vector superfield in constructing the CP^N SNLSM later. The vector superfield is written in the Wess-Zumino gauge as

$$V(y, \theta, \bar{\theta}) = -\theta\sigma^\mu\bar{\theta}A_\mu(y) + i\theta\theta\bar{\theta}\bar{\lambda}(y) - i\bar{\theta}\bar{\theta}\theta^\alpha\left[\lambda_\alpha(y) + \frac{1}{4}\epsilon_{\alpha\beta}C^{\beta\gamma}\sigma_{\gamma\dot{\gamma}}^\mu\{\bar{\lambda}^{\dot{\gamma}}, A_\mu\}\right] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[D(y) - i\partial_\mu A^\mu(y)]. \quad (14)$$

The C -deformed part in the $\bar{\theta}\bar{\theta}\theta$ term is introduced in order that the component fields transform canonically under the gauge transformation. The powers of V are obtained by

$$V^2 = \bar{\theta}\bar{\theta}\left[-\frac{1}{2}\theta\theta A_\mu A^\mu - \frac{1}{2}C^{\mu\nu}A_\mu A_\nu + \frac{i}{2}\theta_\alpha C^{\alpha\beta}\sigma_{\beta\dot{\alpha}}^\mu[A_\mu, \bar{\lambda}^{\dot{\alpha}}] - \frac{1}{8}|C|^2\bar{\lambda}\lambda\right], \quad (15)$$

$$V^3 = 0, \quad (16)$$

where $|C|^2 = C^{\mu\nu}C_{\mu\nu} = 4\det C$.

3 Supersymmetric CP^N Sigma Model on Noncommutative Superspace

The Lagrangian of four-dimensional $\mathcal{N} = 1$ supersymmetric nonlinear sigma model (SNLSM) is written using the Kähler potential K as

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} K(\Phi, \bar{\Phi}). \quad (17)$$

The same expression can be used for $\mathcal{N} = 2$ SNLSM in two dimensions. The results derived below for the four-dimensional case hold true for the two-dimensional case after slight modification. See the argument of dimensional reduction to two dimensions in the end of this section.

In the case of a single pair of chiral and antichiral superfields, the Berezin integration in eq.(17) with the noncommutativity (1) was calculated in [10]. It is given by

$$\mathcal{L} = \mathcal{L}(C = 0) + \sum_{n=1}^{\infty} (\det C)^n [A_n F^{2n-1} + B_n F^{2n}], \quad (18)$$

where A_n and B_n are functions of the component fields. Eq.(18) contains infinitely many terms since generally K is not a polynomial and the powers of θ are nonzero. We have not found a good way to analyze this model as it stands.

Some SNLSMs are expressed as supersymmetric gauge theories. Such SNLSMs contain the model whose target space is a Hermitian symmetric space [12] (*e.g.* CP^N and Grassmannian $G_{N,M}$), T^*CP^{N-2} and $T^*G_{N,M}$. We construct the CP^N SNLSM on noncommutative superspace as the noncommutative extension of [11] using the result of [13]. We start from the following Lagrangian

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} \left[\bar{\Phi}^i * e^V * \Phi^i - V \right], \quad (19)$$

where $i = 1, 2, \dots, N+1$. V is the $U(1)$ vector superfield. It is written modulo total derivatives in terms of component fields as

$$\begin{aligned} \mathcal{L} = & \bar{F}^i F^i - i\bar{\psi}^i \bar{\sigma}^\mu \mathcal{D}_\mu \psi^i - \mathcal{D}_\mu \bar{\phi}^i \mathcal{D}^\mu \phi^i + \frac{1}{2} \bar{\phi}^i D \phi^i + \frac{i}{\sqrt{2}} (\bar{\phi}^i \lambda \psi^i - \bar{\psi}^i \bar{\lambda} \phi^i) \\ & + \frac{i}{2} C^{\mu\nu} \bar{\phi}^i F_{\mu\nu} F^i - \frac{1}{16} |C|^2 \bar{\phi}^i \bar{\lambda} \bar{\lambda} F^i - \frac{1}{\sqrt{2}} C^{\alpha\beta} (\mathcal{D}_\mu \bar{\phi}^i) \sigma_{\beta\dot{\alpha}}^\mu \bar{\lambda}^{\dot{\alpha}} \psi_\alpha^i \\ & - \frac{1}{2} D. \end{aligned} \quad (20)$$

Here \mathcal{D}_μ is the gauge covariant derivative defined by

$$\mathcal{D}_\mu \phi^i = \partial_\mu \phi^i + \frac{i}{2} A_\mu \phi^i, \quad \mathcal{D}_\mu \psi^i = \partial_\mu \psi^i + \frac{i}{2} A_\mu \psi^i. \quad (21)$$

² T^*CP^N denotes the cotangent bundle of CP^N . $T^*G_{N,M}$ is similar.

Following [13], we redefine the antichiral superfields $\bar{\Phi}^i$ in the Lagrangian (20) as

$$\begin{aligned}\bar{\Phi}^i(\bar{y}, \bar{\theta}) &= \bar{\phi}^i(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\psi}^i(\bar{y}) \\ &+ \bar{\theta}\bar{\theta}\left[\bar{F}^i(\bar{y}) + iC^{\mu\nu}\partial_\mu(\bar{\phi}^i A_\nu)(\bar{y}) - \frac{1}{4}C^{\mu\nu}\bar{\phi}^i A_\mu A_\nu(\bar{y})\right],\end{aligned}\quad (22)$$

so that the component fields transform canonically under the gauge transformation.

Eq.(20) contains the auxiliary fields F^i , \bar{F}^i , D , λ , $\bar{\lambda}$ and A_μ . They have the role of imposing constraints on the fields as follows.

$$\bar{F}^i : F^i = 0, \quad (23)$$

$$F^i : \bar{F}^i + \frac{i}{2}C^{\mu\nu}\bar{\phi}^i F_{\mu\nu} - \frac{1}{16}|C|^2\bar{\phi}^i\bar{\lambda}\bar{\lambda} = 0, \quad (24)$$

$$D : \bar{\phi}^i\phi^i = 1, \quad (25)$$

$$\lambda^\alpha : \bar{\phi}^i\psi_\alpha^i = 0, \quad (26)$$

$$\bar{\lambda}^{\dot{\alpha}} : \frac{i}{\sqrt{2}}\bar{\psi}_{\dot{\alpha}}^i\phi^i - \frac{1}{8}|C|^2\bar{\phi}^i\bar{\lambda}_{\dot{\alpha}}F^i - \frac{1}{\sqrt{2}}C^{\alpha\beta}(\mathcal{D}_\mu\bar{\phi}^i)\sigma_{\beta\dot{\alpha}}^\mu\psi_\alpha^i = 0, \quad (27)$$

$$\begin{aligned}A_\mu : \frac{1}{2}\bar{\psi}^i\bar{\sigma}^\mu\psi^i + \frac{i}{2}(\bar{\phi}^i\partial^\mu\phi^i - \partial^\mu\bar{\phi}^i\cdot\phi^i) - \frac{1}{2}(\bar{\phi}^i\phi^i)A^\mu \\ + iC^{\mu\nu}\partial_\nu(\bar{\phi}^iF^i) - \frac{i}{2\sqrt{2}}C^{\alpha\beta}\sigma_{\beta\dot{\alpha}}^\mu\bar{\lambda}^{\dot{\alpha}}\bar{\phi}^i\psi_\alpha^i = 0.\end{aligned}\quad (28)$$

After eliminating F^i and \bar{F}^i , the Lagrangian (20) takes a simple form

$$\mathcal{L} = -\mathcal{D}_\mu\bar{\phi}^i\mathcal{D}^\mu\phi^i - i\bar{\psi}^i\bar{\sigma}^\mu\mathcal{D}_\mu\psi^i, \quad (29)$$

with the constraints

$$\bar{\phi}^i\phi^i = 1, \quad (30)$$

$$\bar{\phi}^i\psi_\alpha^i = 0, \quad (31)$$

$$\bar{\psi}_{\dot{\alpha}}^i\phi^i + iC^{\alpha\beta}\sigma_{\beta\dot{\alpha}}^\mu(\mathcal{D}_\mu\bar{\phi}^i)\psi_\alpha^i = 0, \quad (32)$$

$$A_\mu = i(\bar{\phi}^i\partial_\mu\phi^i - \partial_\mu\bar{\phi}^i\cdot\phi^i) + \bar{\psi}^i\bar{\sigma}^\mu\psi^i. \quad (33)$$

The constraints (30-32) are solved as follows

$$\phi^i = \frac{1}{\sqrt{1+\bar{\varphi}\varphi}} \begin{pmatrix} \varphi^a \\ 1 \end{pmatrix}, \quad \bar{\phi}^i = \frac{1}{\sqrt{1+\bar{\varphi}\varphi}} \begin{pmatrix} \bar{\varphi}^{\bar{a}} \\ 1 \end{pmatrix}, \quad (34)$$

$$\psi_\alpha^i = \frac{1}{\sqrt{1+\bar{\varphi}\varphi}} P^{ij} \chi_\alpha^j, \quad \chi_\alpha^i = \begin{pmatrix} \chi_\alpha^a \\ 0 \end{pmatrix}, \quad (35)$$

$$\bar{\psi}_\alpha^i = \frac{1}{\sqrt{1+\bar{\varphi}\varphi}} \left[\bar{\chi}_\alpha^j P^{ji} - i C^{\alpha\beta} \sigma_{\beta\dot{\alpha}}^\mu \bar{\phi}^i (\partial_\mu \bar{\phi}^j) P^{jk} \chi_\alpha^k \right], \quad \bar{\chi}_\alpha^i = \begin{pmatrix} \bar{\chi}_\alpha^{\bar{a}} \\ 0 \end{pmatrix}, \quad (36)$$

where $a, \bar{a} = 1, 2, \dots, N$. $P^{ij} = \delta^{ij} - \phi^i \bar{\phi}^j$ is a projection operator which satisfies

$$P^2 = P, \quad \bar{\phi}^i P^{ij} = P^{ij} \phi^j = 0. \quad (37)$$

Substituting eqs.(33-36) into eq.(29), the Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_C, \quad (38)$$

$$\mathcal{L}_0 = -g_{a\bar{b}} \partial_\mu \varphi^a \partial^\mu \bar{\varphi}^{\bar{b}} - i g_{a\bar{b}} \bar{\chi}^{\bar{b}} \bar{\sigma}^\mu D_\mu \chi^a - \frac{1}{4} R_{a\bar{b}c\bar{d}} (\chi^a \chi^c) (\bar{\chi}^{\bar{b}} \bar{\chi}^{\bar{d}}), \quad (39)$$

$$\mathcal{L}_C = 2g_{a\bar{b}} g_{c\bar{d}} C^{\alpha\beta} (\sigma^{\mu\nu})_\beta^\gamma \chi_\alpha^a \chi_\gamma^c (\partial_\mu \bar{\varphi}^{\bar{b}}) (\partial_\nu \bar{\varphi}^{\bar{d}}), \quad (40)$$

where $g_{a\bar{b}}$, $D_\mu \chi^a$ and $R_{a\bar{b}c\bar{d}}$ are given by

$$g_{a\bar{b}} = \frac{(1 + \bar{\varphi}\varphi) \delta_{ab} - \bar{\varphi}^{\bar{a}} \varphi^b}{(1 + \bar{\varphi}\varphi)^2}, \quad D_\mu \chi^a = \partial_\mu \chi^a + \Gamma_{bc}^a (\partial_\mu \varphi^b) \chi^c, \quad (41)$$

$$\Gamma_{bc}^a \equiv g^{a\bar{d}} \partial_b g_{c\bar{d}}, \quad R_{a\bar{b}c\bar{d}} \equiv -g_{a\bar{e}} \partial_c (g^{f\bar{e}} \partial_{\bar{d}} g_{f\bar{b}}) = g_{a\bar{b}} g_{c\bar{d}} + g_{a\bar{d}} g_{c\bar{b}}. \quad (42)$$

$g_{a\bar{b}}$ is the Fubini-Study metric of CP^N . Γ_{bc}^a and $R_{a\bar{b}c\bar{d}}$ are the Christoffel symbol and the Riemann curvature tensor respectively. In the CP^1 case, the C -deformed part \mathcal{L}_C vanishes.

$$\mathcal{L}_C^{(CP^1)} = 2(1 + \bar{\varphi}\varphi)^{-4} C^{\alpha\beta} (\sigma^{\mu\nu})_\beta^\gamma \chi_\alpha \chi_\gamma (\partial_\mu \bar{\varphi}) (\partial_\nu \bar{\varphi}) = 0. \quad (43)$$

We study supersymmetry of the Lagrangian (38). In the $C = 0$ case, the $\mathcal{N} = 1$ supersymmetry transformation is generated by Q_α and $\bar{Q}_{\dot{\alpha}}$. Q_α

generates the transformation

$$\delta\varphi^a = \sqrt{2}\xi\chi^a, \quad \delta\bar{\varphi}^{\bar{a}} = 0, \quad (44)$$

$$\delta\chi_\alpha^a = -\sqrt{2}\Gamma_{bc}^a(\xi\chi^b)\chi_\alpha^c, \quad \delta\bar{\chi}_{\dot{\alpha}}^{\bar{a}} = -\sqrt{2}i(\bar{\sigma}^\mu\xi)_{\dot{\alpha}}\partial_\mu\bar{\varphi}^{\bar{a}}. \quad (45)$$

In the present case with $C \neq 0$, the same transformation (44) and (45) give

$$\delta g_{a\bar{b}} = \partial_c g_{a\bar{b}} \delta\varphi^c + \partial_{\bar{c}} g_{a\bar{b}} \delta\bar{\varphi}^{\bar{c}} = \sqrt{2}g_{b\bar{b}}\Gamma_{ac}^b(\xi\chi^c), \quad (46)$$

$$\delta(g_{a\bar{b}}\chi_\alpha^a\partial_\mu\bar{\varphi}^{\bar{b}}) = \left[\sqrt{2}g_{b\bar{b}}\Gamma_{ac}^b(\xi\chi^c)\chi_\alpha^a - \sqrt{2}\Gamma_{bc}^a(\xi\chi^b)\chi_\alpha^c g_{a\bar{b}} \right] \partial_\mu\bar{\varphi}^{\bar{b}} = 0. \quad (47)$$

We then obtain

$$\delta\mathcal{L}_C = \delta\left[2C^{\alpha\beta}(\sigma^{\mu\nu})_{\beta}{}^{\gamma}(g_{a\bar{b}}\chi_\alpha^a\partial_\mu\bar{\varphi}^{\bar{b}})(g_{c\bar{d}}\chi_\gamma^c\partial_\nu\bar{\varphi}^{\bar{d}})\right] = 0. \quad (48)$$

We have shown that the Lagrangian (38) is invariant under the $\mathcal{N} = \frac{1}{2}$ supersymmetry transformation (44) and (45).

Using dimensional reduction, we obtain the Lagrangian of CP^N SNLSM on noncommutative superspace in two dimensions.

$$\begin{aligned} \mathcal{L}_{2D} = & \frac{1}{2}g_{AB}\partial_z\varphi^A\partial_{\bar{z}}\varphi^B + ig_{a\bar{b}}\left(\bar{\chi}_L^{\bar{b}}D_{\bar{z}}\chi_L^a + \bar{\chi}_R^{\bar{b}}D_z\chi_R^a\right) + R_{a\bar{b}c\bar{d}}\chi_L^a\bar{\chi}_L^{\bar{b}}\chi_R^c\bar{\chi}_R^{\bar{d}} \\ & + 2g_{a\bar{b}}g_{c\bar{d}}(C^{11}\chi_L^a\chi_L^c - C^{22}\chi_R^a\chi_R^c)\epsilon^{\mu\nu}(\partial_\mu\bar{\varphi}^{\bar{b}})(\partial_\nu\bar{\varphi}^{\bar{d}}), \end{aligned} \quad (49)$$

where

$$\varphi^A = (\varphi^a, \bar{\varphi}^{\bar{a}}), \quad g_{AB} = \begin{pmatrix} 0 & g_{a\bar{b}} \\ g_{b\bar{a}} & 0 \end{pmatrix}, \quad \chi_\alpha^a = \begin{pmatrix} \chi_L^a \\ \chi_R^a \end{pmatrix}, \quad \bar{\chi}_{\dot{\alpha}}^{\bar{a}} = \begin{pmatrix} \bar{\chi}_L^{\bar{a}} \\ \bar{\chi}_R^{\bar{a}} \end{pmatrix}. \quad (50)$$

4 Discussion

In this letter, we have studied the supersymmetric CP^N sigma model on noncommutative superspace. We have constructed a closed form of the Lagrangian of the model (38). We have found that the $\mathcal{N} = \frac{1}{2}$ supersymmetry transformation law of the model is not modified.

In two dimensions ordinary NLSMs with extended supersymmetry have a few remarkable properties.

- i) Models are integrable (at least at classical level).
- ii) They have good UV divergence properties, *i.e.*, finite to certain loops for $\mathcal{N} = 2$ and finite to all loops for $\mathcal{N} = 4$.
- iii) They possess instantons.

It is interesting to see whether these nice properties hold true for two-dimensional SNLSM on noncommutative superspace.

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